t aspects of the
l. In this case,
e Ising pressure
if J and $dJ/d\sigma$
simply related
69 kT_c) and it
$dJ/d\sigma<0$. Let
ere n is a small
pical disordered-
r a small range

(11)

its.

ure

an external ap-
$_{xt}=p_{dl}+p_I$. We
nal pressure, for
of p_I and $-p_{dl}$
$T_2<\cdots T_6<T_7$.
e isotherms will
xternal pressure
ied (that is, if
nt of p_I). Now
$p_{ext}=0$. As the
σ can increase
to 5 on Fig. 1),
ecomes unstable
must be a first-
further heating
σ_7. However, on
ecrease smoothly
the instability
rst-order change
ses smoothly on
sis loop near the
in σ at T_5 on
T_3 on cooling;
t on Fig. 1. The
um width of this
nically unstable
temperature T_4
equals that at
uilibrium would
nd no hysteresis.
or 4′ and 3′ on
to show that a
letermining T_4 in

schematic sketch
β^T in the critical
ched from *below*,
s to the value B
. On cooling, as
nishes and jumps
the system is in
, $1/\beta^T$ never van-

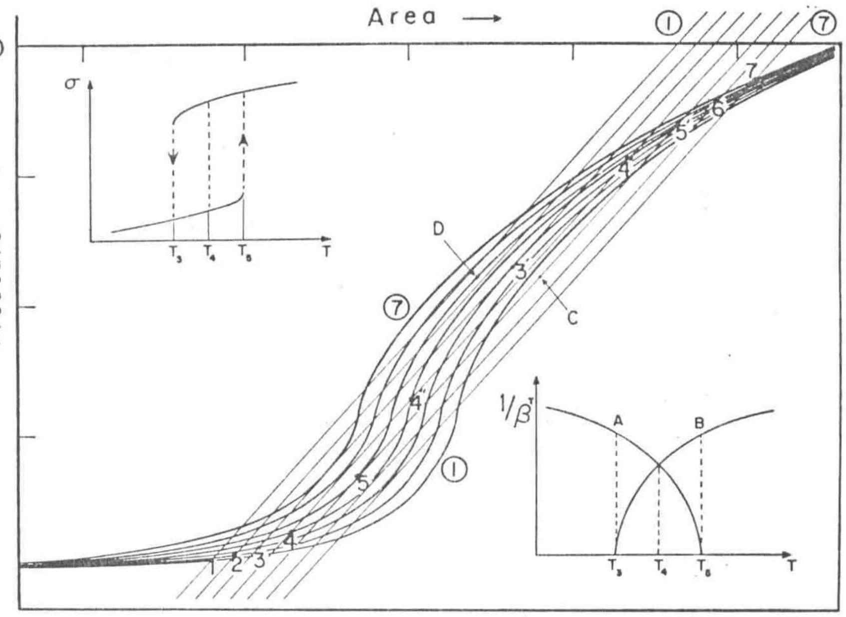

FIG. 1. Behavior of a two-dimensional Ising model as a function of temperature at vanishing external pressure. The family of *curves* p_I were calculated at seven evenly-spaced temperatures from T_1 to T_7. The family of *straight lines*—p_{dl} were drawn to represent a disordered lattice with typical compressibility and thermal expansion coefficients. The encircled numbers 1 and 7 indicate the spin and lattice isotherms at T_1 and T_7. The insets represent schematically the temperature dependences of the area σ and of the reciprocal isothermal compressibility $1/\beta^T$.

ishes but has a singular point at T_4. If the transition occurs in the metastable region but before the mechanical instability point is reached, $1/\beta^T$ will show hysteresis and discontinuities but does not vanish.

If an actual crystal behaves like this model, it is impossible to bring it arbitrarily close to a lambda point: a first-order transition occurs before the temperature reaches the theoretical critical temperature. Indeed, unless great care is taken to achieve true thermodynamic equilibrium, there are a range of temperatures (such as T_3 to T_5 in Fig. 1), where the properties depend on the history of the sample.

Constant Temperature

Let us look at the variation of area σ as a function of the applied pressure p_{ext}. Figure 2 shows, at a given temperature, the Ising pressure and the negative of the disordered-lattice pressure as functions of σ. At zero external pressure, the equilibrium point is at A, which corresponds to a largely disordered system. As the external pressure is increased, the ordering of the system increases and the area σ decreases smoothly until the pressure reaches a value equal to BB' at which mechanical instability occurs. The system spontaneously contracts to an area σ_C corresponding to Point C, which is the new equilibrium state under this external pressure of magnitude $p_3=CC'=BB'$. A further increase in the external pressure causes a smooth decrease of the area and completes the ordering. If the pressure is now reduced, the system is mechanically stable until the area reaches the value $\sigma_D>\sigma_C$. At D the system is mechanically unstable and spontaneously expands to the value σ_E, the new equilibrium area under this pressure of magnitude $p_1=EE'=DD'$. Again the possibility of hysteresis is predicted in a region which corresponds to metastable (or local) equilibrium. If the system were in complete thermodynamic equilibrium a first-order transition without hysteresis would take place at pressure p_2.

Constant Area

If the area is maintained constant by an applied pressure and not by rigid clamping, Inequality (10) is still valid. Therefore on each curve $p_I(T)$ for a given area, there is a forbidden zone in which the intersection of the isochores $-p_{dl}(T)$ and $p_I(T)$ does not correspond to a stable state. On Fig. 3 are plotted several Ising isochores corresponding to areas $\sigma_1<\sigma_2<\cdots\sigma_6<\sigma_7$. The

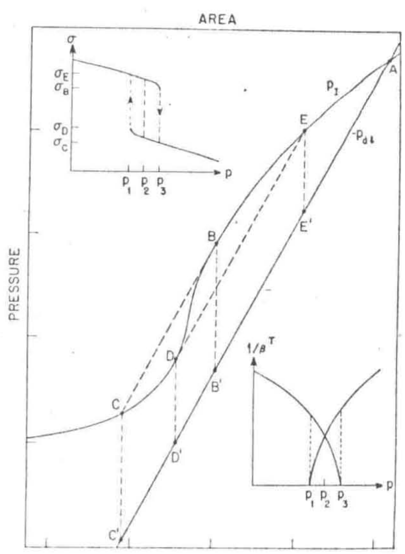

FIG. 2. Behavior of a two-dimensional Ising model as a function of pressure at constant temperature. The insets represent schematically the pressure dependence of the area σ and of the reciprocal isothermal compressibility $1/\beta^T$.